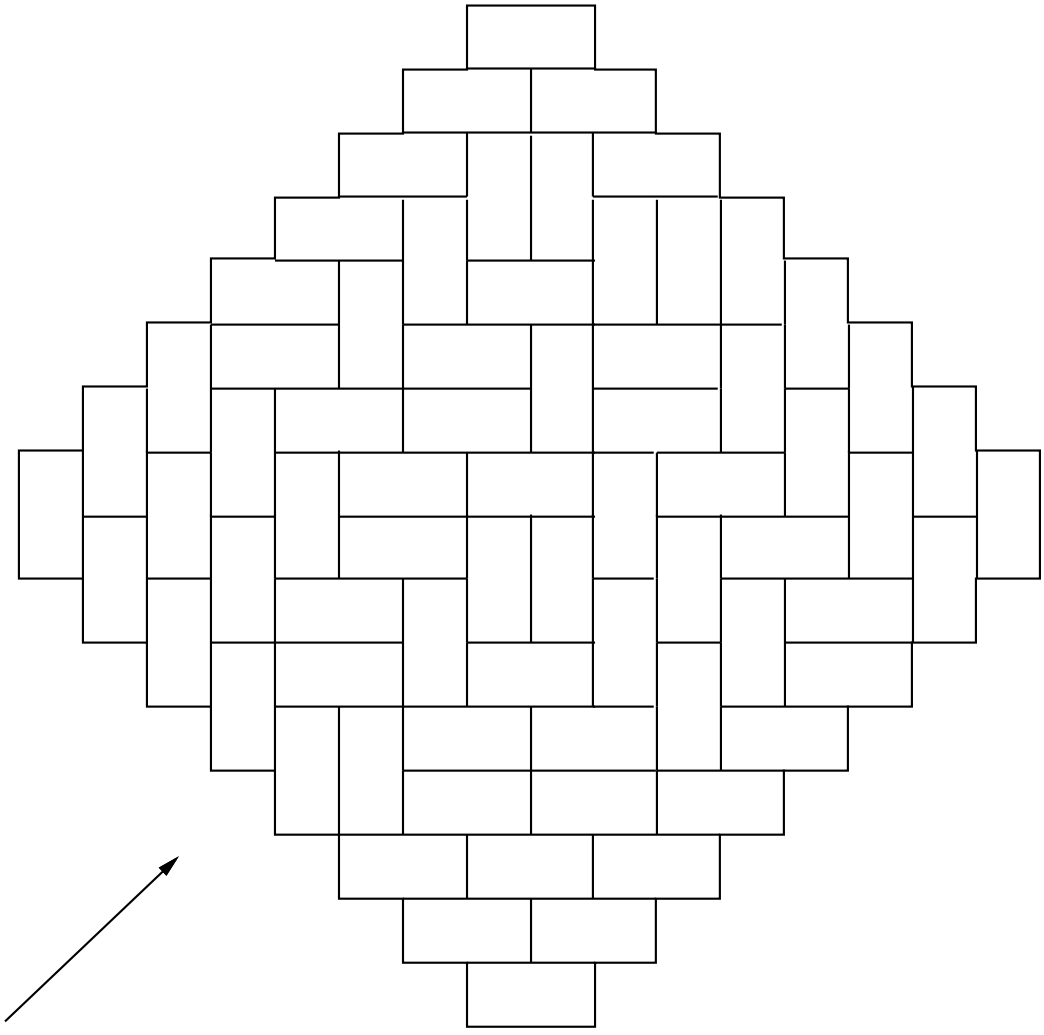
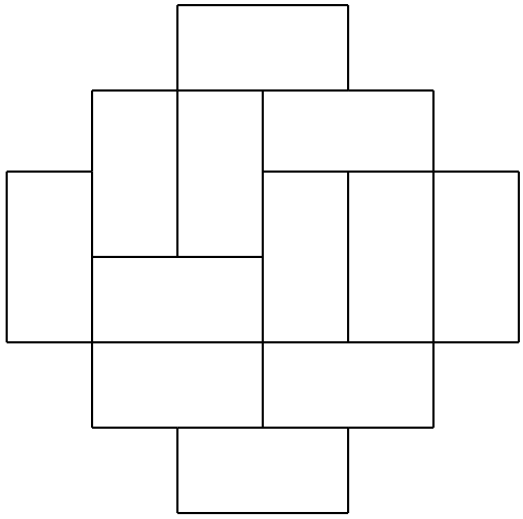
**Theorem (Elkies, Kuperberg, Larsen, and Propp): The Aztec diamond of order $n$ has exactly**

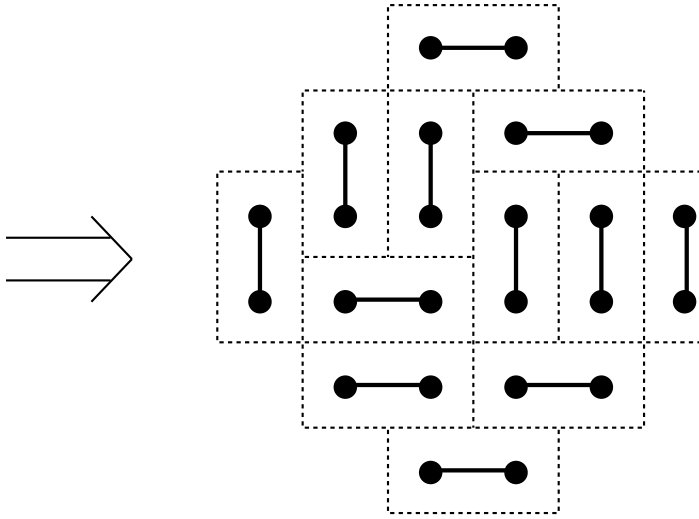
$$2^{n(n+1)/2}$$

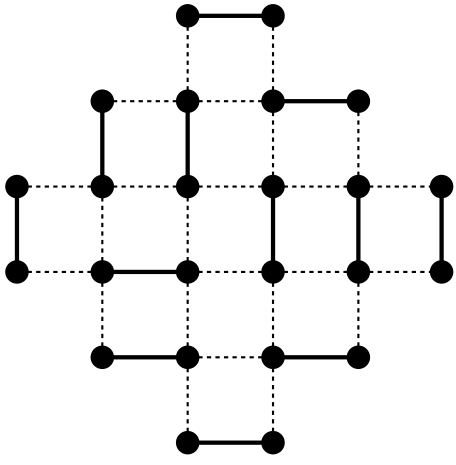
**tilings by dominoes.**

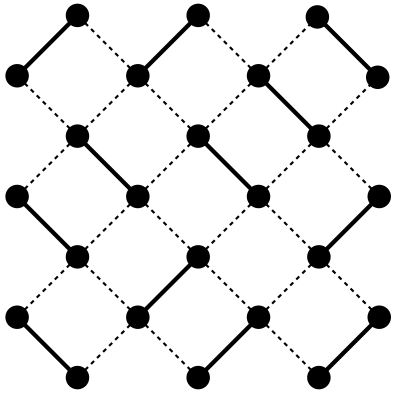


(a domino tiling of the Aztec diamond of order 8)





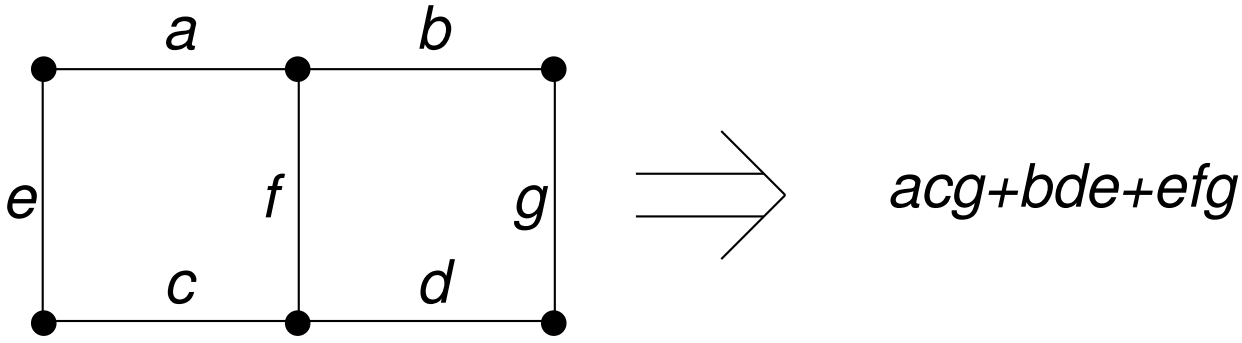




(rotate 45 degrees
for good measure)

**with its edges, we consider the sum of the weigh**

**of all the perfect matchings of *G*. For instance:**



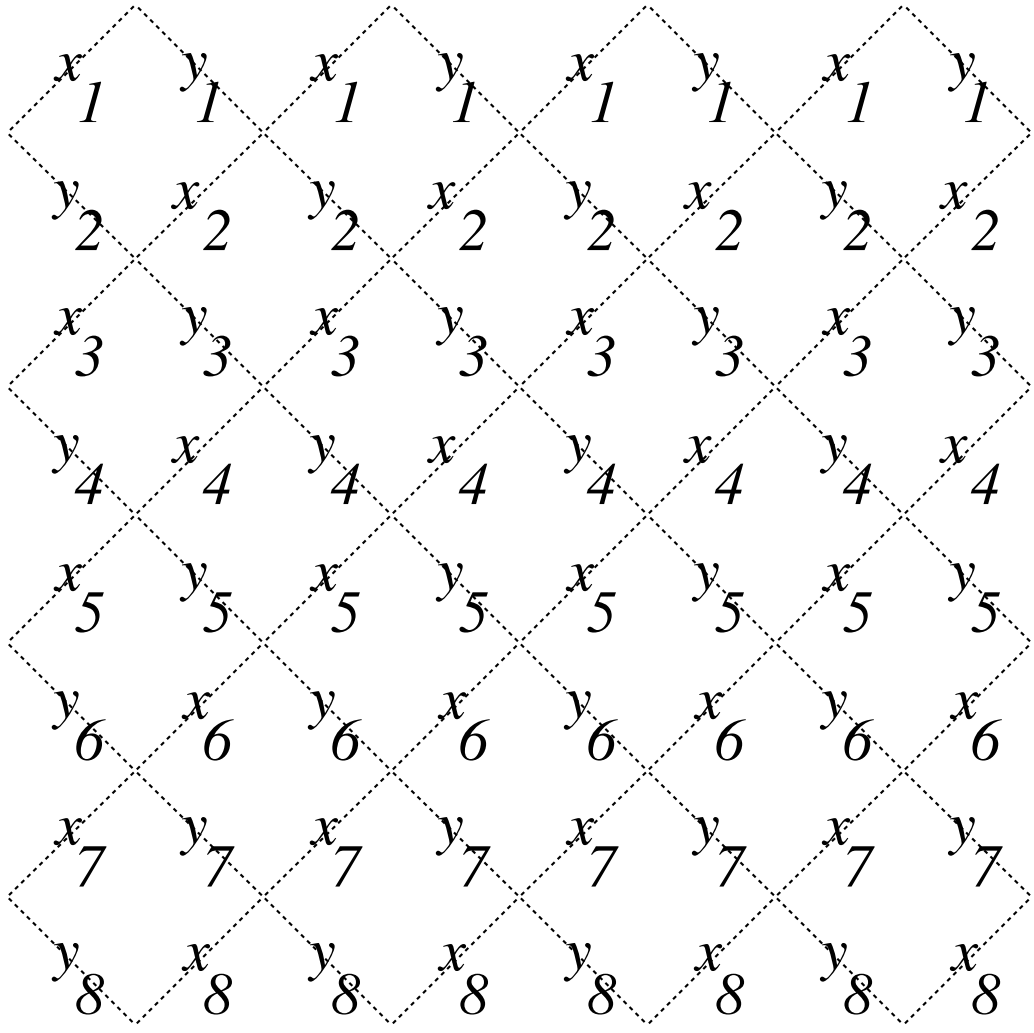

(A *perfect matching* of a graph *G* is a collection of edges of *G* that jointly contain each vertex of *G* exactly once, and the *weight* of a collection of edg is the product of the weights of the edges.)

**The number of domino tilings of a region like th**

**Aztec diamond is the sum of the weights of all t**

**perfect matchings of the dual graph, where eac**

**edge is given weight 1.**


x 1 y 1 x 1 y 1 x 1 y 1 x 1 y 1
y 2 x 2 y 2 x 2 y 2 x 2 y 2 x 2
x 3 y 3 x 3 y 3 x 3 y 3 x 3 y 3
y 4 x 4 y 4 x 4 y 4 x 4 y 4 x 4
x 5 y 5 x 5 y 5 x 5 y 5 x 5 y 5
y 6 x 6 y 6 x 6 y 6 x 6 y 6 x 6
x 7 y 7 x 7 y 7 x 7 y 7 x 7 y 7
y 8 x 8 y 8 x 8 y 8 x 8 y 8 x 8

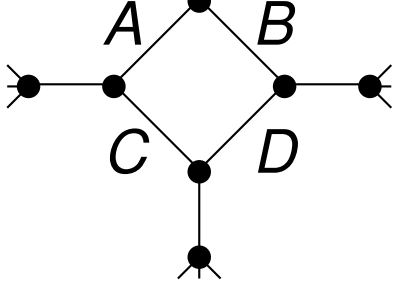
**the sum of the weights of the matchings of the Aztec diamond of order $n$ is**

$$\prod_{1 \le i < j \le n} (x_i x_j + y_i y_j)$$

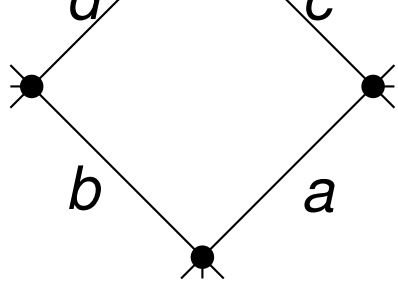
$i$ **odd**

$j$ **even**

The sum of the weights of the matchings of a graph that contains the patch

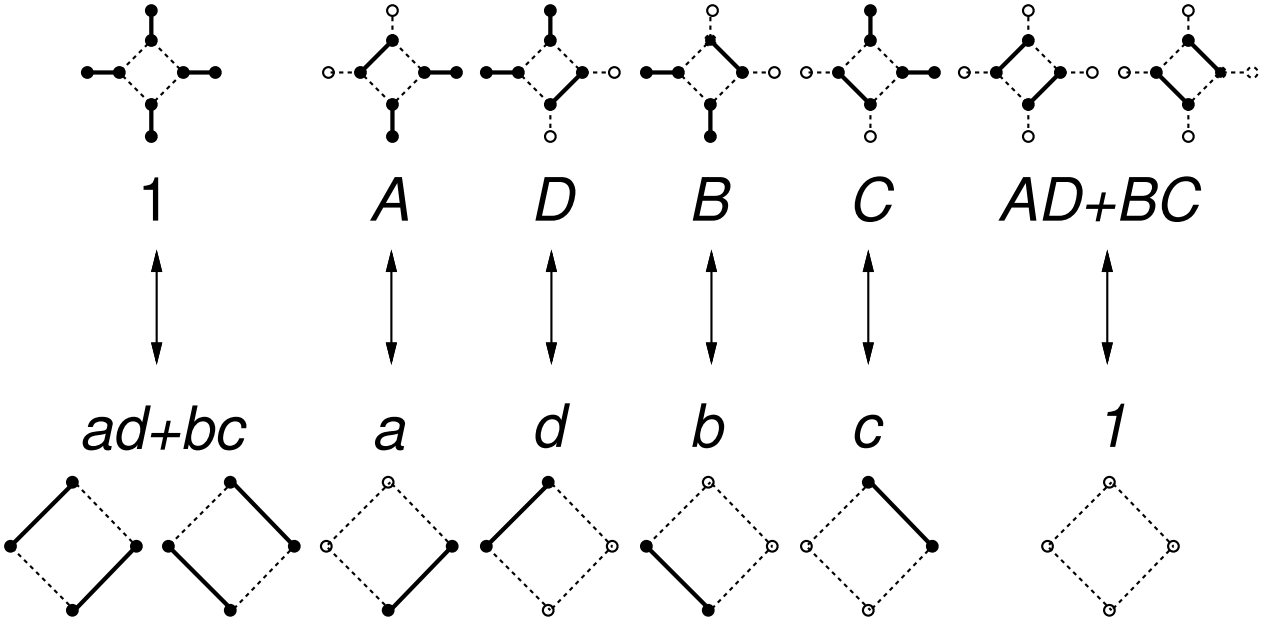



equals $R$ times the sum of the weights of the matchings of the graph obtained by replacing the patch by

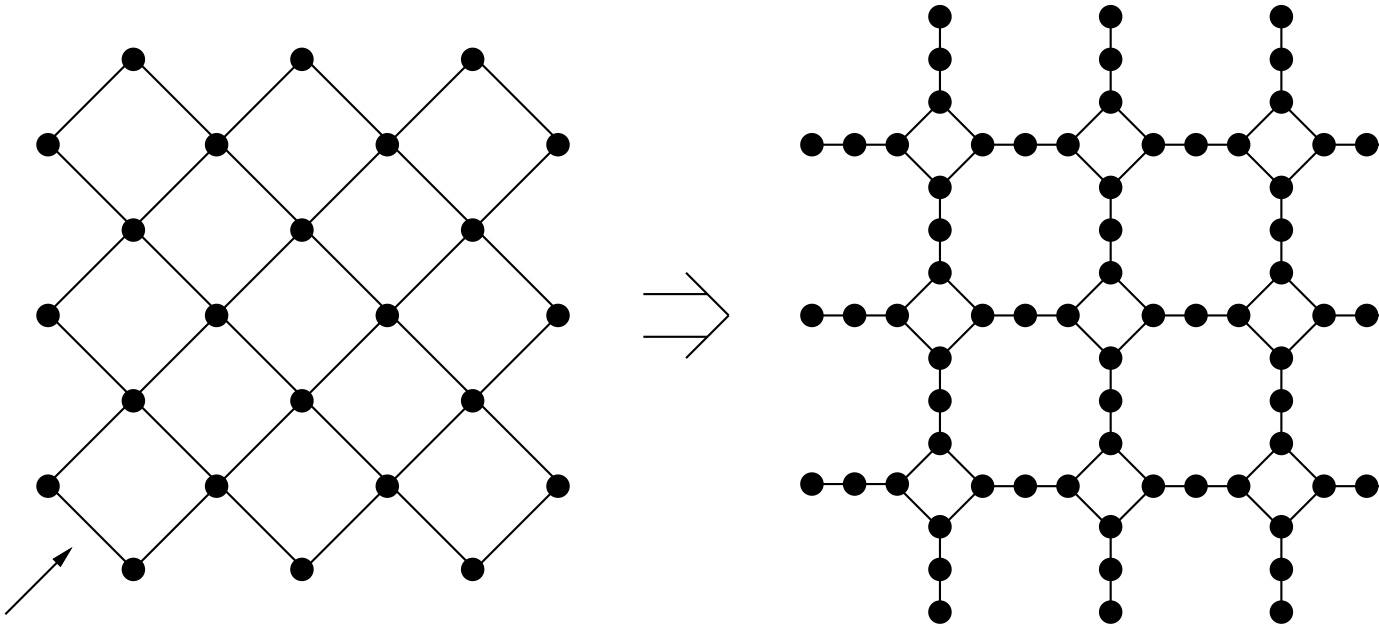



where unmarked edges have weight 1,
$R=AD+BC$, $a=A/R$, $b=B/R$, $c=C/R$, $d=D/R$.

Check that the following two lists of numbers differ by a factor of $R$ (note $ad+bc=1/R$):



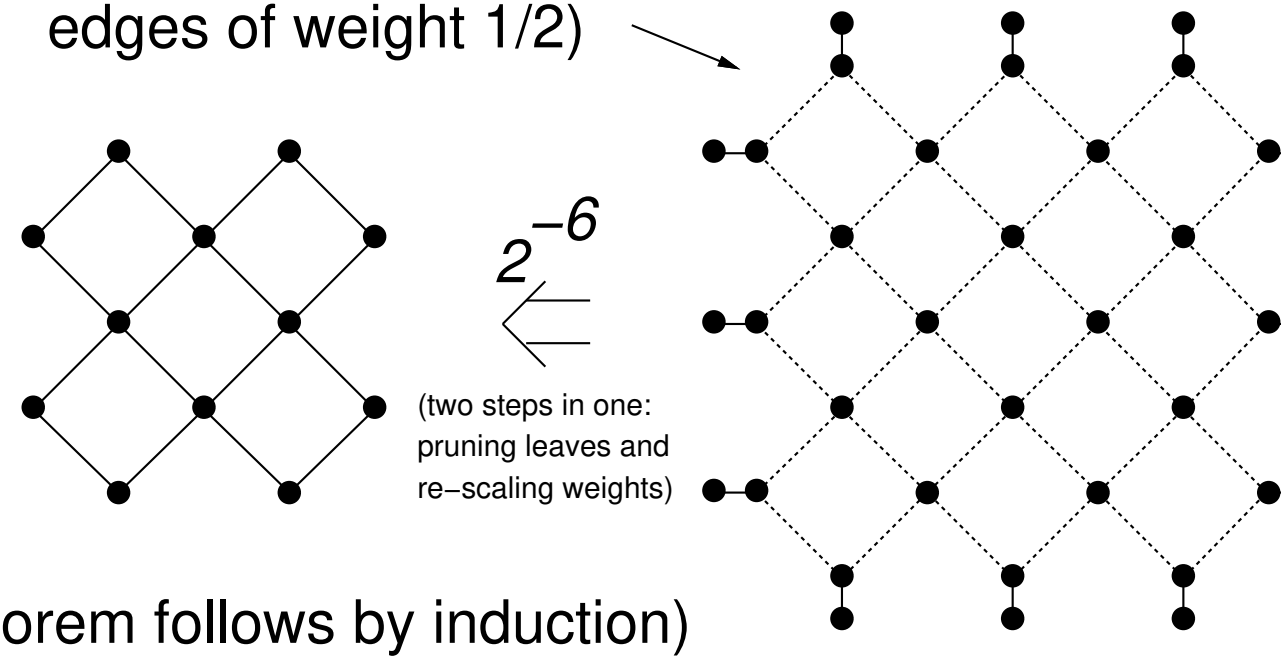
**dual graph of the diamond of order $n-1$ while picking up an extra weight factor of $2^n$.**



(solid lines denote edges of weight 1)

$\Downarrow 2^9$

(dashed lines denote edges of weight 1/2)



$2^{-6}\Leftarrow$

(two steps in one: pruning leaves and re−scaling weights)

(theorem follows by induction)