18.177 Ergodic Theory and Interacting Particle Systems

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Tu-Th 9:30-11:00, room 1-135

Course description

The course will begin with an introduction to measurable dynamics, in which we study iteration of measure-preserving maps from a probability space to itself. In addition to looking at basic notions of recurrence, we will also study ergodicity and mixing and their connection with the spectral properties of the Hilbert-space operator associated with the dynamical system.

The important idea of entropy will then be introduced, leading to a proof of Ornstein's isomorphism theorem. The proof I will present will be my own stream-lined version of the Rothstein-Burton proof, which is not given a satisfactory exposition anywhere in the literature.

We will then consider situations in which we are given a measurable map from a measure-space to itself but are not given a particular measure that is preserved by the map. We will look at properties of the space of invariant measures, and in particular, we will see how the notion of entropy allows us to single out a subset of these measures consisting of those that are "as random as possible". We will see that this approach can be enriched by importing the notion of energy from statistical mechanics, and looking for what are called equilibrium states (probability measures that maximize "pressure"). The connection between equilibrium states and Gibbs states will be explored.

In the final part of the course, I will talk about interacting particle systems, using many ideas from the first part of the course. We will see that the same probability measures that are equilibrium states can also be attractors for suitable interactive dynamics.

Since is the first time I have taught such a course, I may not end up covering all of the preceding material, but I intend to try.

There will be a homework assignment roughly every two weeks. Students taking the course for credit will be expected to attend regularly and to hand in the homework.