# Counting constrained domino tilings of Aztec diamonds

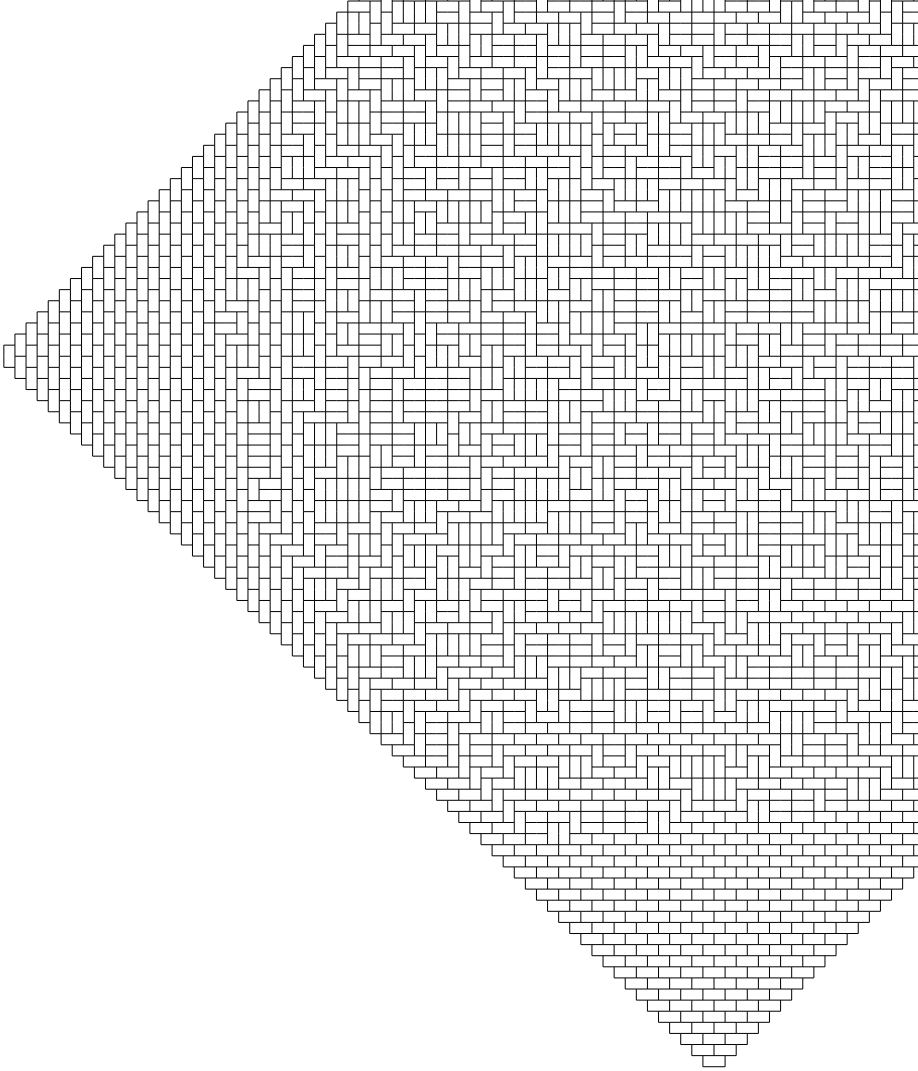
Ira Gessel, Alexandru Ionescu, and James Propp

**Note: The results described in this presentation will appear in several different articles.**

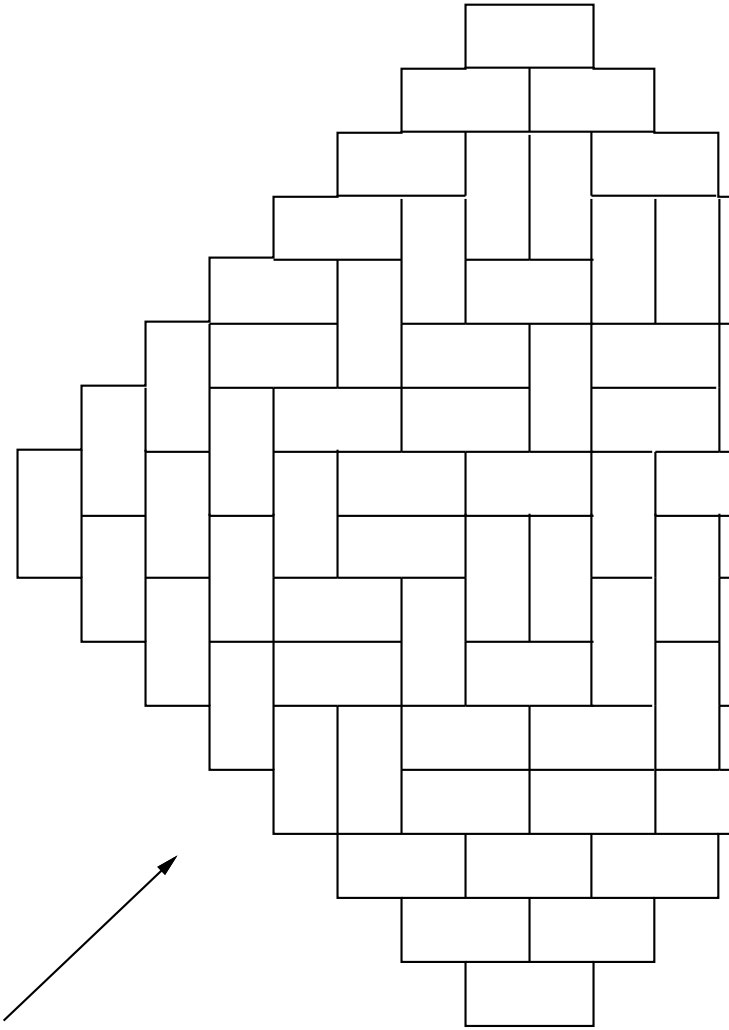
of domino tilings of large Aztec d

Specifically, we define a three–va
function whose coefficients give
probabilities associated with ran
Aztec diamonds, and we show th
function is in fact a rational funct

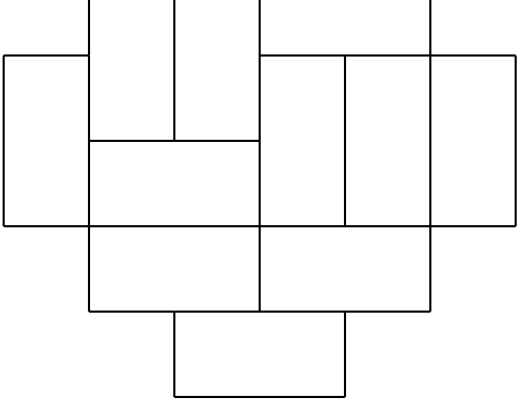
Our proof makes use of a still–my
algorithm called "domino shufflin
into the algebraic significance of
may permit us to devise extensio
to apply these extensions to othe
concerning typical behavior of til

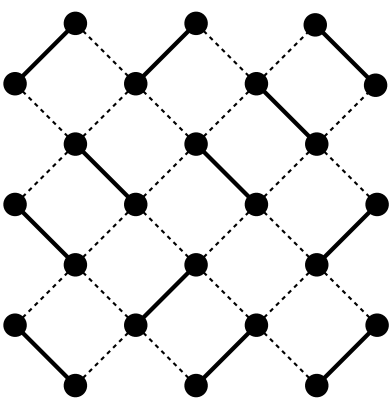


**A typical domino tiling of a large Aztec**

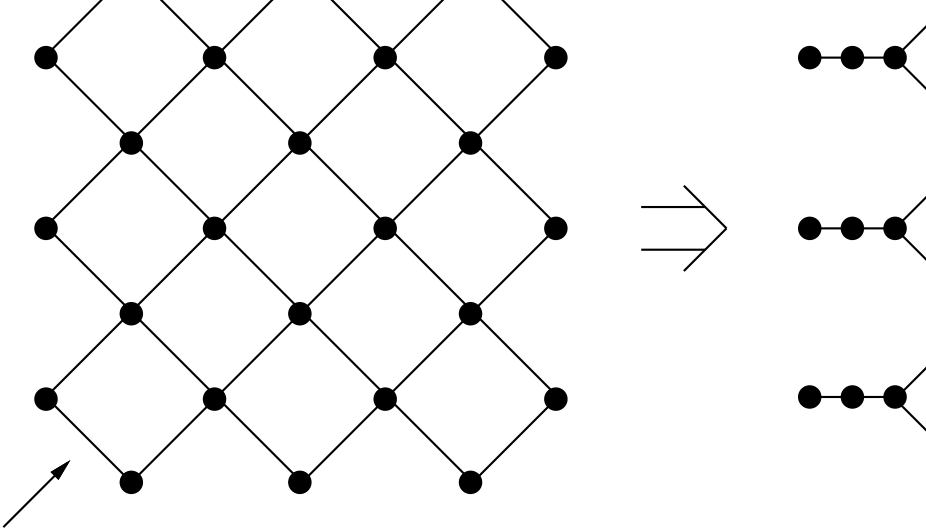


(a domino tiling of the Aztec dia



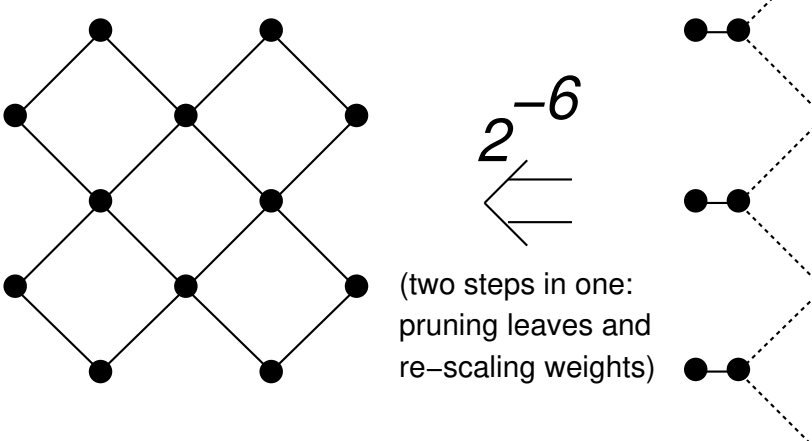


(rotate 45 degrees
for good measure)

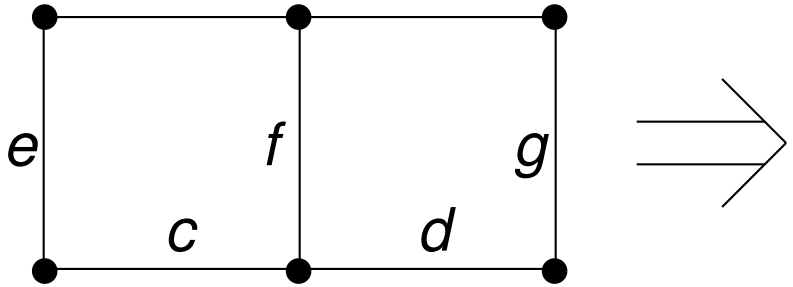


(solid lines denote edges of weight 1)

(dashed lines denote
edges of weight 1/2)




(theorem follows by induction)




(A *perfect matching* of a graph *G* i
edges of *G* that jointly contain eac
exactly once, and the *weight* of a
is the product of the weights of th

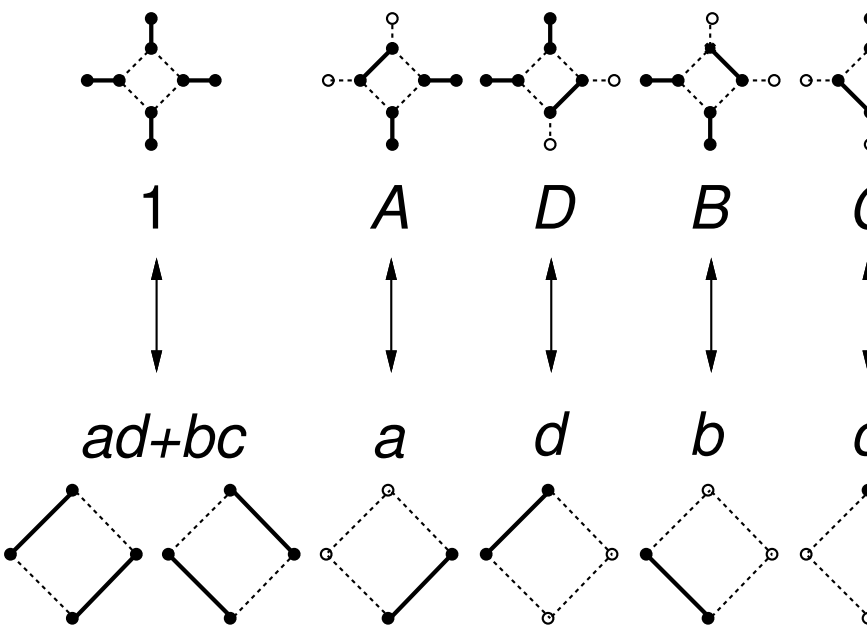
**The number of domino tilings o**
**Aztec diamond is the sum of th**
**perfect matchings of the dual g**
**edge is given weight 1.**

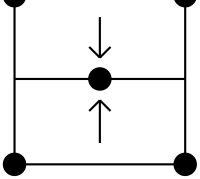
where unmarked edges have wei

$R=AD+BC,\ a=A/R,\ b=B/R,\ c=C/R$

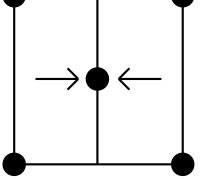
Check that the following two lists

differ by a factor of $R$ (note $ad+bc$



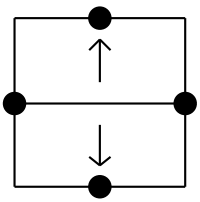


or

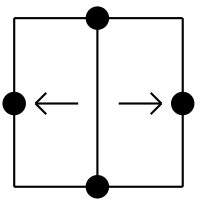


**SLIDE** all remaining dominoes one step in the direction of their

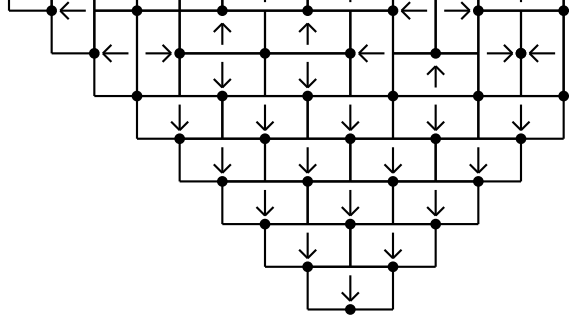
**CREATE** new 2–by–2 blocks of the form



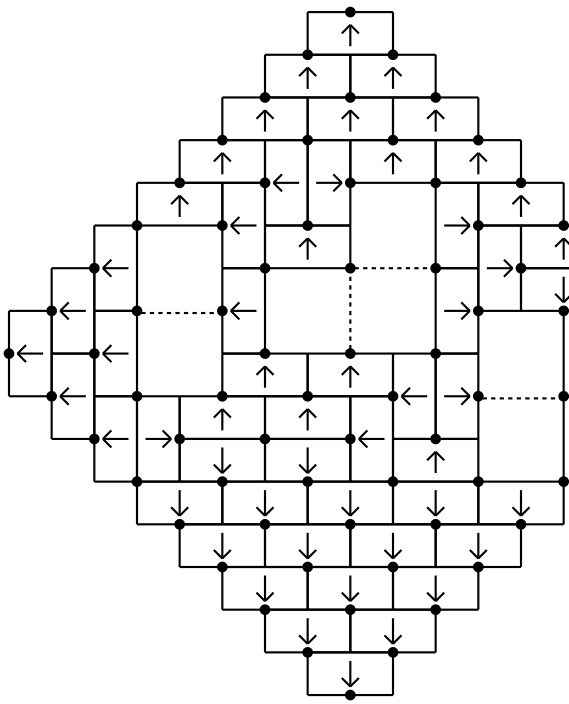
or



in all the new open spaces, using a fair coin to decide

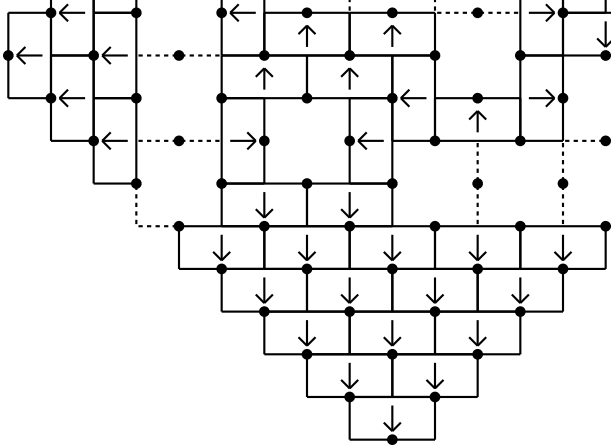


DESTRUCTION

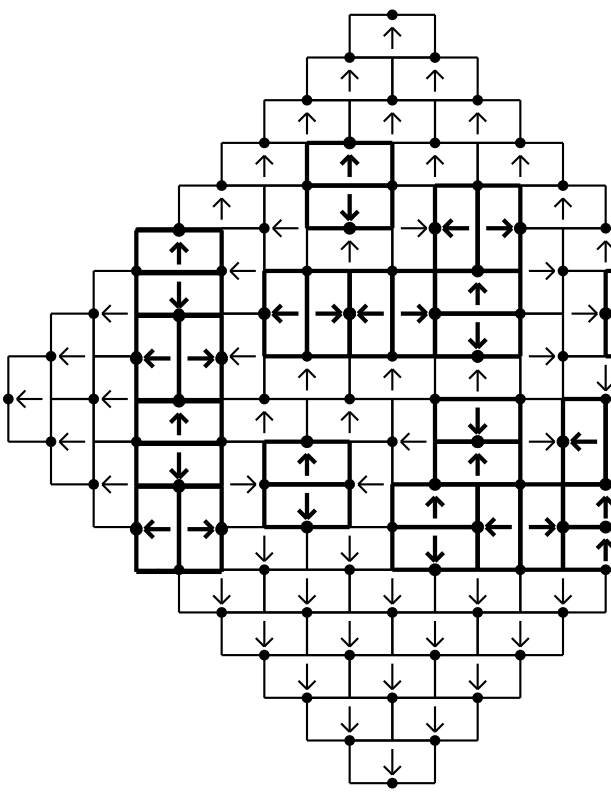


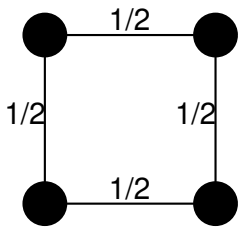
SLIDING

...

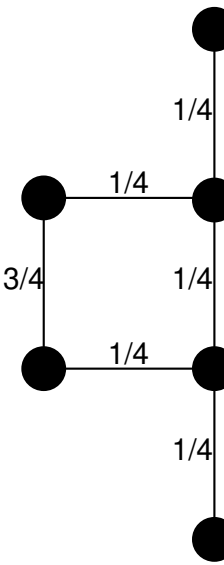


CREATION



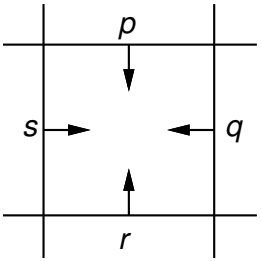



$G_1$

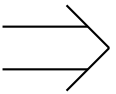


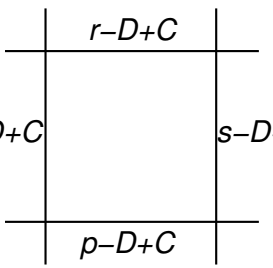

Note that at each vertex of $G_n$, th

probabilities of the incident edges

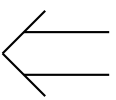



**DESTRUCTION**



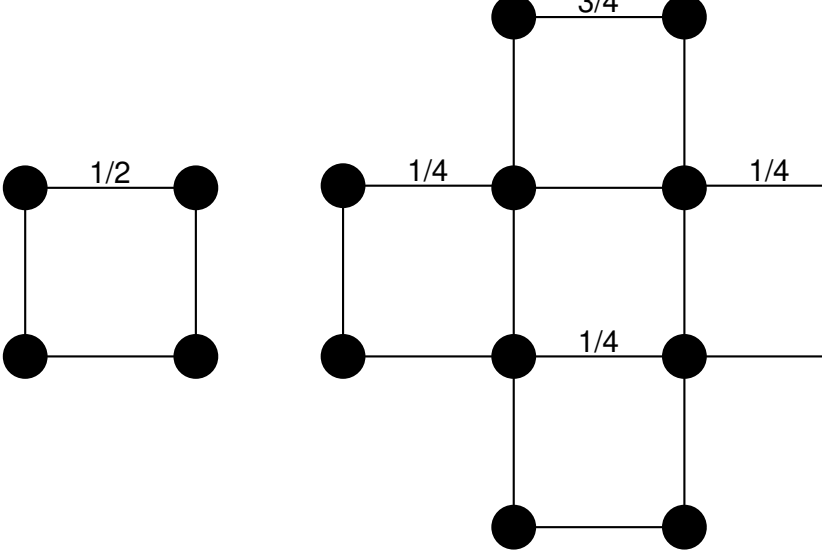



**CREATION**



This allows us to prove that these pr

satisfy a linear recurrence relation.




We encode these via a generating
three variables:

$$\left(1/2\right)\ z + \left( \begin{array}{c} 3/4\ y \\ +\ 1/4\ x^{-1} \qquad +\ 1/4 \\ +\ 1/4\ y^{-1} \end{array} \right.$$

(coefficient of $x^j y^k z^n$ equals probability associated with the edge a
in the Aztec diamond of order $n$ )

**Consequence (Cohn, Elkies, and**

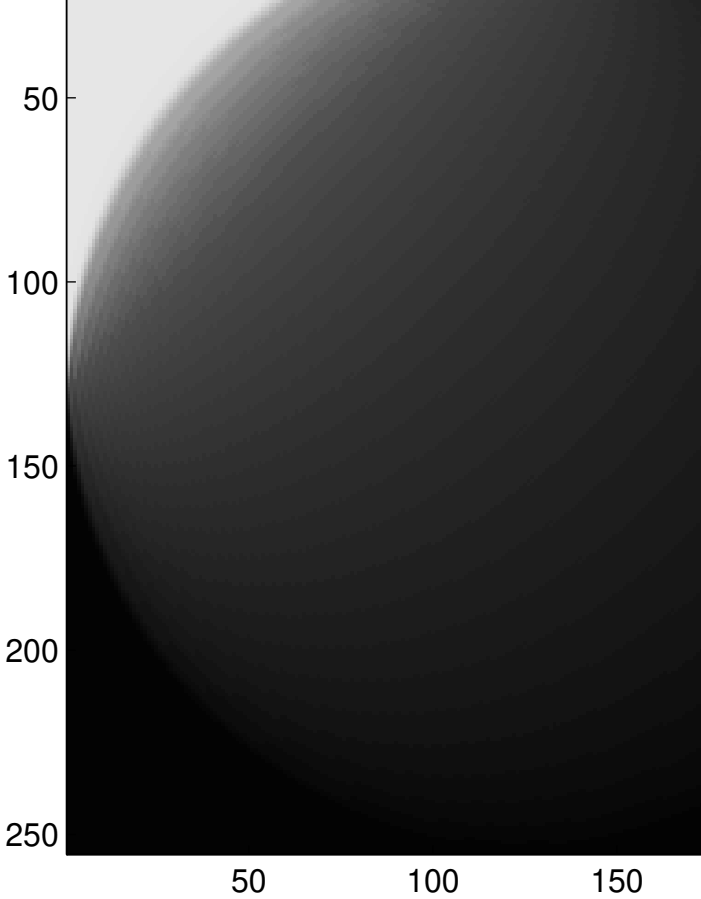
**probability that a randomly chos**

**of the Aztec diamond of order** $n$

**going domino at (normalized) loc**

**(with** $|s|+|t|<1$ **) is asymptotic to**

$$P(s,t) = \begin{cases} 0 & \text{if} \\ \quad \text{(note sharp cut-offs} & \\ 1 & \text{if} \\ \frac{1}{2} + \frac{1}{\pi} \tan^{-1}((2t-1)/\sqrt{1-2s^2-2t^2}) & \text{if} \end{cases}$$


50
100
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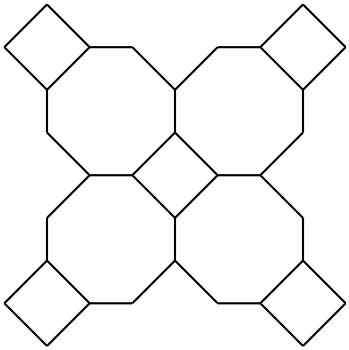
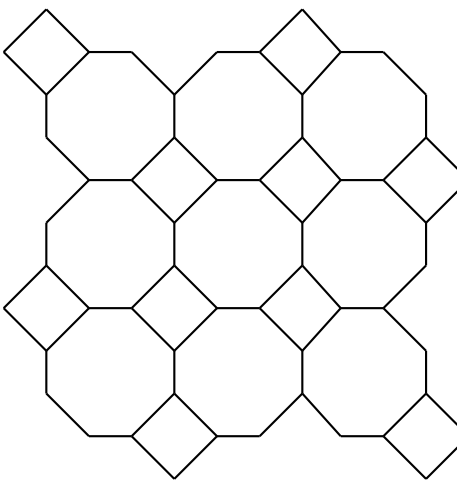
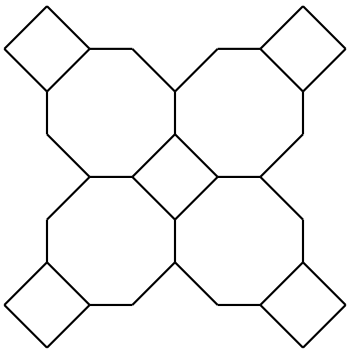




**For each such graph, the numbe**

**matchings is either a power of 5**

**power of 5 (Yang, 1991).**




**However, we have not yet figured**
**of domino shuffling that applies**

**Such an algorithm would give us**
**to the problem of computing the**
**behavior of tilings of these graph**

*Update: Such an algorithm has now*
*For more details, write to propp@m*
*or check out http://www–math.mit.e*