Math 431, Assignment #8: Solutions

(due 4/19/01)

1. Chapter 5, problem 15:

   (a) $P(X > 5) = 1 - P(X < 5) = 1 - \Phi(\frac{5-10}{6}) = 1 - \Phi(-5/6) = 1 - (1 - \Phi(5/6)) = \Phi(5/6) \approx \Phi(.83) \approx .7967$; allowing for round-off errors, a good answer would be .797 or .80.

   (b) $P(4 < X < 16) = \Phi(\frac{16-10}{6}) - \Phi(\frac{4-10}{6}) = \Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1 \approx 2(.8413) - 1 = .6826$, or about .68.

   (c) $P(X < 8) = \Phi(\frac{8-10}{6}) = \Phi(-1/3) \approx 1 - \Phi(.33) = 1 - .6293 = .3707$, or about .37.

   (d) $P(X < 20) = \Phi(\frac{20-10}{6}) = \Phi(5/3) \approx \Phi(1.67) = .9525$, or about .95.

   (e) $P(X > 16) = 1 - \Phi(\frac{16-10}{6}) = 1 - \Phi(1) \approx 1 - .8413 = .1587$, or about .16.

2. Chapter 5, problem 16: $P(X > 50) = P(\frac{X-40}{4} > \frac{10}{4}) = 1 - \Phi(2.5)$ and $P(X \leq 50) = \Phi(2.5) \approx .9938$. It is appropriate to multiply .9938 by itself ten times if we assume that the amount of rainfall in each year is independent of the amount of rainfall in the other years. Hence the desired probability is $(P(X < 50))^{10} \approx (.9938)^{10}$.

3. Chapter 5, problem 19: Letting $Z = (X - 12)/2$, we have Z a standard normal random variable. Now, we want $.10 = P(Z > (c - 12)/2)$. But from Table 5.1, $P(Z < 1.28) \approx .90$ and so $(c - 12)/2 \approx 1.28$ or $c \approx 14.56$.

4. Chapter 5, problem 20: Let X denote the number in favor. Then X is binomial with mean 65 and standard deviation $\sqrt{(100)(.65)(.35)} \approx 4.77$. Let Z be a standard normal random variable.

   (a) $P(X \geq 50) = P(X \geq 49.5) \approx P(\frac{X-65}{4.77} \geq \frac{49.5-65}{4.77}) = P(\frac{X-65}{4.77} \geq \frac{-15.5}{4.77}) \approx P(Z \geq -3.25) \approx .9994$.

(b) $P(59.5 \le X \le 70.5) \approx P(\frac{-5.5}{4.77} \le Z \le \frac{5.5}{4.77}) \approx 2P(Z \le 1.15) - 1 \approx .75$.

(c) $P(X \le 74.5) \approx P(Z \le \frac{9.5}{4.77}) \approx .977$.

5. Chapter 5, problem 23:

   (a) $P(149.5 < X < 200.5) = P(\frac{149.5-(1000)(1/6)}{\sqrt{(1000)(1/6)(5/6)}} < Z < \frac{200.5-1000/6}{\sqrt{(1000)(1/6)(5/6)}}) \approx \Phi(\frac{200.5-166.7}{\sqrt{5000/36}}) - \Phi(\frac{149.5-166.7}{\sqrt{5000/36}}) \approx \Phi(2.87) - \Phi(1.46) - 1 \approx .9258$.

   (b) $P(X < 149.5) = P(Z < \frac{149.5-(800)(1/5)}{\sqrt{(800)(1/5)(4/5)}}) \approx P(Z < -.93) = 1 - \Phi(.93) \approx .1762$.

6. Chapter 5, problem 30:

   (a) $e^{-(1/2)(2)} = e^{-1} \approx .37$.

   (b) $e^{-(1/2)(10)}/e^{-(1/2)(9)} = e^{-1/2} \approx .61$.

7. Chapter 5, problem 31: $e^{-(1/8)(8)} = e^{-1} \approx .37$.

8. Chapter 5, problem 32:

   (a) Since the exponential distribution is memoryless, $P(X > 30|X > 10) = P(X > 20) = e^{-1}$.

   (b) $P(X > 30|X > 10) = \frac{P(X>30)}{P(X>10)} = \frac{1/4}{3/4} = \frac{1}{3}$.

9. Chapter 5, theoretical exercise 12:

   (a) For x between a and b, $F(x) = (x-a)/(b-a)$, which equals $\frac{1}{2}$ when $x = (b+a)/2$.

   (b) For all x, $F(x) = \Phi((x-\mu)/\sigma)$, which equals $\frac{1}{2}$ when $(x-\mu)/\sigma = 0$, i.e., when $x = \mu$.

   (c) For x greater than 0, $F(x) = 1 - e^{-\lambda x}$, which equals $\frac{1}{2}$ when $x = (\ln 2)/\lambda$.