Math 431, Assignment #2: Solutions

(due 2/8/01)

1. (a) Since A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B) = .3 + .5 = .8$. (b) Since A and B are mutually exclusive, the event that A occurs but B doesn't is exactly the same as the event that A occurs, and its probability is .3. (c) Since A and B are mutually exclusive, the probability that A and B both occur is 0.

2. We have $1 \geq P(E \cup F) = P(E) + P(F) - P(EF)$. Now just re-arrange the inequality $1 \geq P(E) + P(F) - P(EF)$.

3. (a) We have $P(A) \leq P(A \cup B) \leq P(A) + P(B)$, so $P(A \cup B)$ must lie between $P(A) = .5$ and $P(A) + P(B) = .9$. Both extremes are feasible: if B is a subset of A then the former happens, whereas if A and B are disjoint then the latter happens. We have $0 \leq P(A \cap B) \leq P(B)$, so $P(A \cap B)$ must lie between 0 and $P(B) = .4$. Both extremes are feasible: if A and B are disjoint then the former happens, whereas if B is a subset of A the latter happens. (b) We have $P(B) \leq P(A \cup B) \leq 1$, so $P(A \cup B)$ must lie between $P(B) = .6$ and 1. Both extremes are feasible: if A is a subset of B then the former happens, whereas if A and B are mutually exhaustive (i.e., their union is the whole sample space) then the latter happens. We have $P(A) + P(B) - 1 \leq P(A \cap B) \leq P(A)$, so $P(A \cap B)$ must lie between $P(A) + P(B) - 1 = .1$ and $P(A) = .5$. Both extremes are feasible: if A and B are mutually exhaustive then the former happens, whereas if A is a subset of B the latter happens.

4. Let R and N denote the events, respectively, that the student wears a ring and that the student wears a necklace. (a) $P(R \cup N) = 1 - 0.6 = 0.4$. (b) $0.4 = P(R \cup N) = P(R) + P(N) - P(RN) = 0.2 + 0.3 - P(RN)$, so $P(RN) = 0.1$.

5. Use the hint: $P(M \cup W \cup G) = P(M) + P(W) + P(G) - P(MW) - P(MG) - P(WG) + P(MWG) = .312 + .470 + .525 - .086 - .042 - .147 + .025 = 1.057 > 1$. Contradiction!

6. There are $\binom{52}{13}$ bridge-hands, but only $\binom{32}{13}$ bridge-hands in which none of the 20 ten-or-higher cards is chosen. Hence the probability of a Yarborough is $\binom{32}{13}/\binom{52}{13} = \frac{32!}{13!19!}/\frac{52!}{13!39!} = 32!39!/52!19! = 5394/9860459 = .0005470$. Since this is less than $.001 = 1/1000$, the Earl was right.

7. The total number of ways for the instructor to choose 5 questions out of the 10 is $\binom{10}{5}$. (a) The number of ways for the instructor to choose 5 questions from the student's preferred 7 is $\binom{7}{5}$. Hence the probability that the student will get 5 preferred problems is $\binom{7}{5}/\binom{10}{5} = 21/252 = 1/12$. (b) The number of ways for the instructor to choose 4 questions from the student's preferred 7 and 1 question from the remaining (unpreferred) 3 is $\binom{7}{4}\binom{3}{1}$, so the number of ways for the test to end up with 4 or more of the student's preferred questions is $\binom{7}{5} + \binom{7}{4}\binom{3}{1} = 21 + 105 = 126$, so the probability of the test ending up with 4 or more of the student's preferred questions is $126/252 = 1/2$.

8. The probability of my winning two matches in a row is equal to the sum of the probabilities of three disjoint events: I win the first two matches but lose the third, I lose the first but win the last two, or I win all three. If I play Gail-Hilda-Gail this gives $\frac{1}{3}\frac{2}{3}\frac{2}{3} + \frac{2}{3}\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3}\frac{1}{3}$ or $10/27$, while if I play Hilda-Gail-Hilda my chance of success is $\frac{2}{3}\frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3}\frac{2}{3} + \frac{2}{3}\frac{1}{3}\frac{2}{3}$ or $8/27$. Hence I am better off playing the stronger player twice.

9. Assume order does not matter. Then the total number of outcomes is $\binom{2K}{4}$, corresponding to the four captured raccoons. The number of outcomes in which there are 2 of each sex is $\binom{K}{2}\binom{K}{2}$. Hence the probability is getting 2 of each sex is $\binom{K}{2}^2/\binom{2K}{4} = (K!/2!(K-2)!)^2/(2K!/4!(2K-4)!) = \frac{K(K-1)}{2}\frac{K(K-1)}{2}\frac{24}{(2K)(2K-1)(2K-2)(2K-3)} = \frac{3K(K-1)}{2(2K-1)(2K-3)}$. Writing this as $\frac{3-3/K}{8-16/K+6/K^2}$ and sending K to infinity, we get $\frac{3}{8}$. This makes sense, because this is exactly the probability of getting two males and two females if the outcomes were completely independent, and the bigger the population is, the less correlation there is between the sex of one randomly chosen individual and the sex of another.