Math 431, Assignment #6

(due 3/29/01)

This assignment covers sections 4.8 and 4.9 (and some earlier material). Most of the problems come from pages 173-188 of Ross.

Remember, as on the previous assignments, to indicate how many hours you spent on the assignment, and whom you worked with. Also remember to show your work if you want to receive credit.

1. Let X be a random variable and t some real number. What value of t minimizes $E((X-t)^2)$, and for this value of t, what is $E((X-t)^2)$?

2. Let X be a binomial random variable with parameters n and p, and let α be some positive real number. What is the expected value of α^X? (Hint: I know two good ways to solve this. One way uses the binomial theorem. The other way uses the fact that a binomial random variable can be written as a sum of independent Bernoulli random variables.)

3. Let V be the number of times you need to throw a die to get a "1" three times. Let W be the number of times you need to throw a die to get a "1" just once.

   (a) Find $P(V = 6 | V > 5)$.

   (b) Find $P(V = 41 | V > 40)$.

   (c) Find $P(W = 6 | W > 5)$.

   (d) Find $P(W = 41 | W > 40)$.

   (e) Explain how you could have predicted that $P(V = 6 | V > 5)$ is far smaller than $P(V = 41 | V > 40)$ before computing them.

   (f) Explain how you could have predicted the relationship that holds between $P(W = 6 | W > 5)$ and $P(W = 41 | W = 40)$ before computing them.

4. Let X be a negative binomial random variable with parameters (r, p). What value of p maximizes $P(X = k)$, for k between 0 and r? (This is the maximum likelihood estimator of p given that the hypergeometric random variable X takes the value k.)

5. Problem 52.

6. Problem 53.

7. Problem 55.

8. Problem 57.

9. Theoretical Exercises, Problem 18.

10. Theoretical Exercises, Problem 25.

Each problem is worth 10 points. Additionally, you can get up to 5 bonus points for making a good estimate of your raw score (which will lie between 0 and 100).