

Math 491, Problem Set #5
(due 9/25/03)

Let a_n be the number of domino tilings of a 3-by- $2n$ rectangle, and let b_n be the number of domino tilings of a 3-by- $(2n + 1)$ rectangle from which a corner square has been removed. We showed in class that $a_n = a_{n-1} + 2b_{n-1}$ and $b_n = a_{n-1} + 3b_{n-1}$ for all $n \geq 1$.

1. Introduce

$$A(x) = a_0 + a_1x + a_2x^2 + \dots$$

and

$$B(x) = b_0 + b_1x + b_2x^2 + \dots$$

Write down two algebraic relations between $A(x)$ and $B(x)$ that represent the two recurrence relations (taking care to incorporate the boundary conditions correctly), and solve for $A(x)$ and $B(x)$.

2. We also saw in class that

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}^n \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Use linear algebra to derive a formula for a_n .

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with.