Math 475, Problem Set #4
(due 2/16/06)

A. Chapter 3, problem 30.

B. (a) Given a row of 25 seats, in how many ways can one choose four non-overlapping blocks, each consisting of four consecutive seats?

(b) What if we require that there be at least one empty seat between any two blocks?

C. (a) The number of linear permutations of the set $\{1, 2, 3, 4, 5, 6\}$ is equal to 3! times 3! times the number of linear permutations of the multiset $\{o, o, o, e, e, e\}$, since we can replace the o's by the three odd numbers in 3! different ways and the e's by the three even numbers in 3! different ways. Verify this numerically by computing the number of linear permutations of the set $\{1, 2, 3, 4, 5, 6\}$ and the number of linear permutations of the multiset $\{o, o, o, e, e, e\}$. (You need not list them all; just count them using formulas from the chapter.)

(b) Is it also true that the number of circular permutations of the set $\{1, 2, 3, 4, 5, 6\}$ is equal to 3! times 3! times the number of circular permutations of the multiset $\{o, o, o, e, e, e\}$? As part of your answer, you should numerically compute the number of circular permutations of the set $\{1, 2, 3, 4, 5, 6\}$ and the number of circular permutations of the multiset $\{o, o, o, e, e, e\}$. Briefly explain what is going on.

(c) Note that the number of linear permutations of the set $\{1, 2, 3, 4, 5, 6\}$ is equal to 6 times the number of circular permutations of the set $\{1, 2, 3, 4, 5, 6\}$. Is it also true that the number of linear permutations of the multiset $\{o, o, o, e, e, e\}$ is equal to 6 times the number of circular permutations of the multiset $\{o, o, o, e, e, e\}$? Briefly explain what is going on.

D. Chapter 3, problem 37.

E. How many different poker hands are there that contain two pairs (of two different ranks) and a fifth card of a different rank than the other four cards?

F. Twelve students taking a potions class are to be divided into four working-groups of three students apiece. All we care about are which students are working together and which students are not. In how many ways can the students be divided into the groups? Be sure to relate your answer to the answer to problem F from the previous assignment.