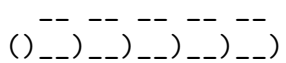
Math 475, Problem Set #13
(due 5/4/06)

A. A baton is divided into five cylindrical bands of equal length, as shown (crudely) below.



In how many different ways can the five bands be colored if n colors are available, and unlimited repetition of the colors is allowed? (Two colorings count as the same if one of them can be converted into the other by turn the baton around.)

B. A circular necklace contains nine beads. Each bead is featureless, so that the necklace has no front or back, and we may flip the necklace over.

(a) How many different necklaces can be constructed if an unlimited number of red and white beads are available?

(b) How many different necklaces can be constructed from three white beads and six red beads?

C. Let S be the set of ways of assigning 12 identical balls to 3 distinguishable boxes, in such a way that no box is empty, and let G be the group of operations on S that permute the boxes. (If it helps, you can imagine that the boxes are different colors. For instance, if $s \in S$ is the assignment that puts 5 balls in the red box, 4 balls in the white box, and 3 balls in the blue box, and $g \in G$ is the element of G that switches the red box and the white box, then gs or $g(s)$ is the assignment that puts 4 balls in the red box, 5 balls in the white box, and 3 balls in the blue box.)

(a) How many objects does S contain?

(b) How many permutations does G contain?

(c) How many orbits does S have under the action of G?

(d) Compare your answer to (c) with the value of $p_3(12)$, the number of partitions of 12 into 3 positive integers. (You can compute $p_3(12)$ however you like).

(If it's unclear what I'm asking for, request clarification!)

D. Assume that an upside down I looks exactly the same as a right-side up I. How many rotationally distinct ways to decorate the faces of a cube with the letter I? (Note that, although there are two ways to color each face, this is NOT the same as asking how many rotationally distinct ways there are to color the faces of a cube with two colors!)

E. In how many rotationally distinct ways can the vertices of a cube be labeled if each vertex is labeled with either a 0 or a 1?