Math 192r, Problem Set #6 (due 10/9/01)

- 1. For each even integer $n \ge 2$, we can represent each domino tiling of a 3-by-*n* rectangle by a code (a_1, a_2, \ldots, a_n) , where a_k is the number of vertical dominos in the *k*th column (always either 0 or 1). Note that two different tilings can have the same code; e.g., for n = 2 there are three tilings but only two codes (namely (0,0) and (1,1)). Formulate a conjecture for the number of codes that occur for general n.
- 2. Let a_n be the number of domino tilings of a 4-by-*n* rectangle, with $n \ge 0$ (we put $a_0 = 1$ by convention).
 - (a) Prove that the sequence a_0, a_1, \ldots satisfies a linear recurrence relation of order 16 or less.
 - (b) Prove that the sequence a_0, a_1, \ldots satisfies a linear recurrence relation of order 8 or less.
 - (c) Prove that the sequence a_0, a_1, \ldots satisfies a linear recurrence relation of order 6 or less.