## Math 192r, Problem Set \#6

(due 10/9/01)

1. For each even integer $n \geq 2$, we can represent each domino tiling of a 3 -by- $n$ rectangle by a code $\left(a_{1}, a_{2}, \ldots, a_{n}\right)$, where $a_{k}$ is the number of vertical dominos in the $k$ th column (always either 0 or 1 ). Note that two different tilings can have the same code; e.g., for $n=2$ there are three tilings but only two codes (namely $(0,0)$ and $(1,1)$ ). Formulate a conjecture for the number of codes that occur for general $n$.
2. Let $a_{n}$ be the number of domino tilings of a 4 -by- $n$ rectangle, with $n \geq 0$ (we put $a_{0}=1$ by convention).
(a) Prove that the sequence $a_{0}, a_{1}, \ldots$ satisfies a linear recurrence relation of order 16 or less.
(b) Prove that the sequence $a_{0}, a_{1}, \ldots$ satisfies a linear recurrence relation of order 8 or less.
(c) Prove that the sequence $a_{0}, a_{1}, \ldots$ satisfies a linear recurrence relation of order 6 or less.
