Math 192r, Problem Set #3 (due 9/26/01)

- 1. Let  $F_n$  be the *n*th Fibonacci number, as Wilf indexes them (with  $F_0 = F_1 = 1, F_2 = 2$ , etc.). Give a simple homogeneous linear recurrence relation satisfied by the sequence whose *n*th term is
  - (a)  $nF_n$ ;
  - (b)  $1F_1 + 2F_2 + \ldots + nF_n$ ;
  - (c)  $nF_1 + (n-1)F_2 + \ldots + 2F_{n-1} + F_n;$
  - (d)  $F_n$  when n is odd, and  $2^n$  when n is even.

In each case, an explanation should be included.

- 2. The sequence of polynomials  $f_n(x)$  in problem 2 of problem set 1 satisfies a second-order linear recurrence relation with coefficients that are Laurent polynomials in x.
  - (a) Find it, and prove that it is correct. (Note that this proves your conjectures from parts (a) through (c) of that problem.)
  - (b) Express  $\sum_{n=0}^{\infty} f_n(x)y^n$  as a rational function of x and y.

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with.