## Math 192r, Problem Set \#3

(due $9 / 26 / 01$ )

1. Let $F_{n}$ be the $n$th Fibonacci number, as Wilf indexes them (with $F_{0}=$ $F_{1}=1, F_{2}=2$, etc.). Give a simple homogeneous linear recurrence relation satisfied by the sequence whose $n$th term is
(a) $n F_{n}$;
(b) $1 F_{1}+2 F_{2}+\ldots+n F_{n}$;
(c) $n F_{1}+(n-1) F_{2}+\ldots+2 F_{n-1}+F_{n}$;
(d) $F_{n}$ when $n$ is odd, and $2^{n}$ when $n$ is even.

In each case, an explanation should be included.
2. The sequence of polynomials $f_{n}(x)$ in problem 2 of problem set 1 satisfies a second-order linear recurrence relation with coefficients that are Laurent polynomials in $x$.
(a) Find it, and prove that it is correct. (Note that this proves your conjectures from parts (a) through (c) of that problem.)
(b) Express $\sum_{n=0}^{\infty} f_{n}(x) y^{n}$ as a rational function of $x$ and $y$.

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with.

