

Math 192r, Problem Set #19
(due 12/6/01)

1. In problem #3 of assignment #17, multivariate polynomials

$$D(x_1, x_3, \dots, x_{2n+1}; y_2, y_4, \dots, y_{2n})$$

were defined. Find an infinite acyclic directed graph with special vertices $\dots, v_{-1}, v_0, v_1, \dots$ where all edges are assigned weight 1 and vertices are assigned weights according to some scheme that you must devise, so that for all integers $i \leq j$ the sum of the weights of the paths from v_i to v_j is $D(x_{2i+1}, x_{2i+3}, \dots, x_{2j+1}; y_{2i+2}, y_{2i+4}, \dots, y_{2j})$. Include a proof that your answer is correct.

2. Consider an infinite array with tilted upper boundary like the one shown below:

$$\begin{array}{cccccccc}
 & & & & & & & \vdots \\
 & & & & & & & x_5 \\
 & & & & & & x_4 & w_5 & y_5 \\
 & & & & x_3 & & w_4 & y_4 & * \\
 & & x_2 & & w_3 & & y_3 & * & * & * \\
 & x_1 & & w_2 & & y_2 & * & * & * & * \\
 w_1 & & y_1 & & * & & * & * & * & * \\
 \vdots & & & & & & & & & \vdots
 \end{array}$$

Here the entries w_i, x_i, y_i are formal indeterminates, and the entries marked with asterisks are determined by the diamond rule as in assignment #17; that is, whenever the array contains four entries arranged like

$$\begin{array}{cc}
 & a \\
 b & & c \\
 & d
 \end{array}$$

we must have $ad - bc = 1$. Some experimentation will probably convince you that each entry in the table is a Laurent polynomial in the variables w_i, x_i, y_i , and that moreover each coefficient in this polynomial equals +1. Show how for each such Laurent polynomial, the Laurent monomials that participate correspond to the perfect matchings of some graph

(just as was the case in assignment #17). Give a concrete description of the graphs and the correspondence between matchings and monomials (including either a proof or convincingly large examples).